Iterative Tuning Strategy for Setting Phase Splits in Traffic Signal Control

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Abstract—This paper introduces Iterative Tuning (IT) strategy for urban traffic signal control. This strategy is motivated by people’s daily repetitive travel patterns between homes and working places. Statistical analysis of a real traffic network shows that traffic flows of junctions are repetitive with small variations on a weekly basis. The main idea of IT is that, daily traffic signal schedules are tuned with anticipation of traffic demands. In this paper, only phase split is tuned iteratively to balance the traffic demands from all directions in a junction. Each junction has its own controller and these controllers can work cooperatively to improve the network performance after several iterations. Therefore IT strategy is scalable for arbitrary large urban networks. Marina Bay and Clementi areas in Singapore based on real traffic data are simulated and simulation results show that IT strategy can improve the performance considerably comparing with fixed-time strategy.

I. INTRODUCTION

Traffic signal control strategies have gone through various development from fixed-time to adaptive strategies, from single junction to coordinated multi-junctions. The fixed-time algorithm was developed by Webster [1] to optimize splits and cycle length with delay estimated model. MAXBAND, developed by Little [2], considers the synchronization of traffic signals so that a car, starting at a main artery and traveling with free speed, can go through several junctions without stop for a red light. TRANSYT (TRAffic Network StudY Tool), developed by Robertson [3], is the most well-known and frequently applied fixed-time traffic signal strategy. It is the benchmark to test the performance improved by adaptive traffic-responsive strategies.

SCATS and SCOOT are the most popular adaptive coordinated traffic strategies. SCATS (Sydney Coordinated Adaptive Traffic System) [4], is a model-free distributed strategies with predefined signal plans. SCOOT (Split Cycle Offset Optimisation Technique) [5], is almost like adaptive TRANSYT strategy with three kinds of optimizer: Split Optimizer, Offset Optimizer and Cycle length Optimizer. OPAC (Optimized Policies for Adaptive Control) [6] strategy is a real-time distributed signal optimization algorithm with three control layers to optimize cycle, split, offset and phase sequences. RHODES (Real-time Hierarchical Optimizing Distributed Effective System) is an adaptive traffic control systems developed by P. Mirchandani et al [7] by using conservation model to predict traffic dynamics. Dynamic programming is employed for OPAC and RHODES to make them not real-time feasible for large-scale traffic networks. Back Pressure (BP) was proposed [8] in traffic signal control and it leads to maximum network throughput with global optimality. Pressure releasing policy [9] extended it with finite queue capacities.

Store-and-forward model of traffic dynamics was first suggested by Gazis and Potts [10], and extended by Dynamics Systems and Simulation Laboratory (DSSL) [11]. Optimization process is carried out with linear quadratic regulator (LQR) based on linear traffic model. After that, de Oliveira [12] reported that considerable improvements might be made by replacing LQR procedures with Centralized Model Predictive Control (CMPC) since MPC takes constraints of phases and links’ maximum volumes into account. Facing large-scale dynamic systems, Lucas Barcelos et al. [13] proposed multi-agent model predictive control to decompose a centralized model predictive control system into decoupled subsystems. However in the analytic model, the topology of network, like turning ratios, is assumed to be constant, which is not correct in reality. As time spreading, analytic model will accumulate errors which leads to suboptimal traffic signal settings. Wang et al. [14] proposed FRIDE to describe traffic dynamics to improve the accuracy of short-term prediction.

In this paper, Iterative Tuning (IT) strategy is motivated by repetitive activities on working days. In urban traffic control system, people travel from homes to workspaces in the morning and return in the evening on working days. Festin [15] studied the general daily traffic profiles in the United States and five areas for the period 1970-1995. Results show that daily traffic profiles are repetitive on a weekly basis. Roess et al. [16] showed that typical variations of daily traffic patterns were around 13% and 16% on weekdays and weekends, respectively.

IT strategy is adapted from Iterative Learning Control (ILC). ILC approach had been applied for density control of freeway traffic flow and achieved robust performance [17]. Huang [18] proposed an iterative learning approach for signal control in urban traffic networks. It uses traffic assignment model to find the global optimal signal control and flow patterns. In control process, traffic signal is controlled to drive the traffic pattern approaching the desired flow pattern. This paper proposes IT strategy to tune phase splits based on repetitive traffic flows. Different from traditional ILC theory, there is no desired trajectory since it is difficult to
find the accurate traffic flow patterns based on any analytic model. The rest of this paper is organized as follows: Section II shows that historical traffic flow patterns are repetitive. Section III describes objectives and tuning processes of IT strategy. Section IV presents two case studies and their simulations. Section V analyzes the simulation results. Section VI concludes this paper and suggests some topics for further research.

II. REPETITIVE TRAFFIC FLOW PATTERNS

For urban traffic system, the most frequently used methodology to describe signalized junctions in United States is introduced in Highway Capacity Manual (HCM) [19]. Before IT strategy is introduced, conceptual frameworks concerning historical traffic flows and IT strategy are summarized.

1) Daily Traffic Pattern (DTP): DTP is the profile of traffic flows along 24 hours a day. From the analysis of Chrobok et al. [20], DTPs of working days are quite similar since activities are almost the same except working days before holidays or weekends, in particular the afternoon’s traffic flows may differ. Therefore, working days are categorized into two classes:

- Normal Working Days: working days except days before holidays.
- Last Working Days: working days before holidays.

2) Daily Traffic Signal Schedules (DTSS): DTSS are phase durations for a junction from 0:00 to 24:00. For a junction, if DTPs of phases are repetitive, traffic demands in all phases are repetitive and repetitive phase durations can be applied. Therefore, DTSS will be identical for the dates with repetitive DTPs of phases.

The basis for IT strategy is the repeatability of DTP. There is an assumption made here: urban infrastructures include road, residences, workplaces etc, changes slowly.

To verify this assumption, historical traffic data of Marina Bay Area, located at Central Business District in Singapore, are analyzed. Fig. 1 shows the network diagram.

![Fig. 1. Marina Bay Area in Singapore](image)

For junction $j$ and $j \in J$, where $J$ is the set of junctions in the network, traffic flows $x_{j,i}(\tau)$ are collected, where $i$ is the index of lane groups; $\tau$ is the index of time intervals. Referring to [16], lane group is defined for one or more lanes approaching one junction that have the same green phase. For every day, there exists one set of traffic flows $x_{j,i}^{\omega}(\tau)$, where $\omega$ is the index of working days; and $\omega \in W$ where $W$ is the set of working days. Mean $M$ in Eq. (1), Standard Deviation $S$ in Eq. (2) and Coefficient of Variation $V$ in Eq. (3) are utilized to analyze the variations.

$$M = \frac{1}{n_{\omega}} \sum_{\omega \in W} x_{j,i}^{\omega}(\tau)$$

$$S = \sqrt{\frac{1}{n_{\omega} - 1} \sum_{\omega \in W} (x_{j,i}^{\omega}(\tau) - M)^2}$$

$$V = \frac{S}{M}$$

where $n_{\omega}$ is number of working days.

In statistic, Coefficient of Variation(CV) expressed in percentage, shows the extent of variability relative to mean value. If CV of traffic flows within certain time interval is very small, traffic flows are almost the same and DTP is repetitive. In this paper, CV is used to indicate the repetition of DTPs for every time interval.

There are 14 junctions included and Junction 2, 5 and 9 are eliminated due to most data errors of loop counts. Daily traffic patterns of class of Normal Working Days for other 11 junctions are analyzed. Time interval $T$ is 15 minutes. Shown in Fig. 2, different junctions have different daily traffic patterns. A day is categorized into Light Traffic Period (00:00 to 07:00) and Heavy Traffic Period (07:00 to 24:00). Traffic signal control system mainly focuses on Heavy Traffic Period. As indicated in Fig. 3, CVs of junctions are lower than 15% during Heavy Traffic Period.

![Fig. 2. Mean values for 11 Junctions](image)

![Fig. 3. Coefficients of variations for 11 Junctions](image)

III. ITERATIVE TUNING STRATEGY FOR PHASE SPLITS

In IT strategy, daily traffic signal schedules are obtained based on daily traffic patterns correspondingly. Each class of daily traffic patterns has its own daily traffic signal schedules. IT strategy has the following features:

- It requires little system knowledge and analytic model of system is not required. Therefore errors from model descriptions are not critical.
- It is an off-line tuning methodology and thus the online computation time is not an issue.
- Repetitive disturbances can be rejected and repetitive errors can be compensated.
- It adapts to the changes of traffic patterns iteratively.

The flow chart of IT strategy is shown in Fig. 4.
A. Data Analysis and Processing

For each junction, historical traffic flow patterns $X_h$ of the entire day are memorized. For junction $j$, $x_{j,i}(k) \in X$, $\forall i \in F_j$ is collected during the entire day, where $X$ represents new-collected traffic flows; $F_j$ is the set of lane groups of junction $j$; where $k$ is the index of cycles. If there is a special event on one day, traffic flows may be different. Therefore, Pattern Checking Algorithm is presented to check whether new-collected traffic flows $X$ are repetitive with any class of historical traffic flow patterns $X_h$ or not.

1) Pattern Checking Algorithm: Pearson product-moment correlation coefficient $\gamma$ is calculated to measure the linear correlation between $X$ and $X_h$. $\gamma^{th}$ is set as the threshold Pearson coefficient. If $\gamma \geq \gamma^{th}$, traffic patterns $X$ are repetitive with historical traffic flow patterns $X_h$. If $\gamma < \gamma^{th}$, traffic patterns $X$ are non-repetitive.

2) Pattern Updating Algorithm: For both classes of working days, historical traffic flows are updated in Eq. (4) to adapt to the slowly changing traffic conditions.

$$X_h = \begin{cases} \alpha X_h + \beta X, & \text{if } \gamma \geq \gamma^{th}; \\ X_h, & \text{if } \gamma < \gamma^{th}. \end{cases}$$

where $\alpha$ and $\beta$ are weighting coefficients and satisfy $\alpha + \beta = 1$, which control the updating speed and do not affect the performance of IT strategy.

The updated traffic flows $X_h$ are stored in the database to be historical traffic flow patterns $X_h$, which are also the inputs of IT strategy.

B. Iterative Tuning

Based on HCM 2000 methodology [19], the least delay time for a single junction is obtained when phase occupancies are balanced. The objective of IT strategy is to minimize the delay time. IT strategy is a decentralized methodology and each junction has its own IT controller. The controllers tune phase splits iteratively until phase occupancies are balanced. Least delay time for each junction is minimized, and delay time of network is reduced.

For junction $j \in J$, based on updated traffic flows $X_h$, traffic flows $X_j(k)$ for all lane groups are retrieved. For a phase $p \in P_j$ with phase duration $u_{j,p}(k)$, where $P_j$ is the set of phases of junction $j$, there are one or more corresponding lane groups $x_{j,i}(k)$ having the right of way, where $x_{j,i}(k) \in X_j(k)$ with $i \in F_j$, and $F_j$ is the set of lane groups of junction $j$ which have the right of way during phase $p$. Maximum phase occupancy $o_{j,p}(k)$ is calculated as the maximum ratio of traffic flows per lane $\frac{x_{j,i}(k)}{n_{i}^{p}}$ and road capacities $s_{u_{j,p}(k)}$, which is expressed in Eq. (5),

$$o_{j,p}(k) = \frac{\max_{i \in F_j,p} \frac{x_{j,i}(k)}{n_{i}^{p}}}{s_{u_{j,p}(k)}}, \quad \forall p \in P_j, \forall j \in J, \forall k \quad (5)$$

where $n_{i}^{p}$ is number of lanes in the lane group $i$; $s$ is the saturation flow per lane and it is assumed to be fixed.

After that, phase occupancy $o_{j,p}(k)$ is converted into phase occupancy error $e_{j,p}(k)$ in Eq. (6). As in Eq. (7), if $e_{j,p}(k) \in E_j(k), \forall p \in P_j$ approaches 0, phase occupancies for this junction are balanced and delay time is almost minimized. $E_j(k)$ is the vector to represent phase occupancy errors of junction $j$.

$$e_{j,p}(k) = o_{j,p}(k) - \frac{\sum_{p \in P_j} o_{j,p}(k)}{\eta^p} \quad (6)$$

$$e_{j,p}(k) \to 0, \quad \forall p \in P_j, \forall j \in J, \forall k \quad (7)$$

where $\eta^p$ is the number of phases for the junction. Assuming $E_j(k)$ is obtained by Eq. (5) and Eq. (6), historical signal duration $U_{j,h}(k)$ is retrieved from the database of traffic signal schedules $U_h$. The tuning procedure for phase splits is constructed below in Eq. (8):

$$\hat{U}_j(k) = U_{j,h}(k) + LE_j(k + 1)$$

$$U_j(k) = C \{ \hat{U}_j(k) \}, \quad \forall j \in J, \forall k \quad (8)$$

where $L$ is defined as the tuning function and $L = \lambda I$; $I$ is the identity matrix with appropriate dimension; $\lambda$ can be determined by trial-and-error method, which may not affect the performance; $C\{}$ is the function to take constraints into account.

For phase $u_{j,p}(k) = C\{ \hat{u}_{j,p}(k) \}$, with $u_{j,p}(k) \in U_j(k), \forall p \in P_j$, constraints are considered. As shown in Eq. (9), function $C\{}$ takes phase constraints into consideration. For the concern of safety, maximum phase time $u_{max}$ and minimum phase time $u_{min}$ for each phase are predefined.

$$u_{min} < u_{j,p}(k) < u_{max}, \quad \forall p \in P_j, \forall j \in J, \forall k \quad (9)$$

Equation Eq. (9) can be realized by:

$$C\{ \hat{u}_{j,p}(k) \} = \begin{cases} \hat{u}_{j,p}(k), & \text{if } u_{min} < \hat{u}_{j,p}(k) < u_{max}; \\ u_{max}, & \text{if } \hat{u}_{j,p}(k) \geq u_{max}; \\ u_{min}, & \text{if } \hat{u}_{j,p}(k) \leq u_{min}. \end{cases} \quad (10)$$

Function $C\{}$ also considers the constraint of cycle length in Eq. (11).

$$\sum_{p \in P_j} u_{j,p}(k) + t_L = C, \quad \forall j \in J, \forall k \quad (11)$$
Where \( C \) is the cycle length; \( t_L \) is the total lost time within a cycle.

This constraint in equation Eq. (11) is satisfied by

\[
C \left\{ \hat{u}_{j,p}(k) \right\} = \frac{\hat{u}_{j,p}(k)}{\sum_{p \in \mathcal{P}} \hat{u}_{j,p}(k)} (C - t_L)
\] (12)

Equations Eq. (10) and Eq. (12) are calculated alternately until the constraints Eq. (9) and Eq. (11) are satisfied simultaneously. Besides, phase durations are set to be accurate to the resolution of controllers.

As phase durations \( U_j(k) \in U, \forall j \in J, \forall k \) are obtained finally, where \( U \) is the vector of phase durations for the entire day, they will be applied for the next date with repetitive traffic flows, which is stored in the database as traffic signal schedules \( U_h \).

IV. CASE STUDIES

In this section, two case studies are introduced. Real traffic flows are simulated based on average values of one month’s raw data.

A. Simulation Platforms

Vissim [21], a microscopic simulator (Developed by PTV Planung Transport Verkehr AG in Karlsruhe, Germany), is used to construct traffic dynamics. In order to program traffic control algorithms easily and efficiently, Vissim is co-simulated with Matlab by COM Interface [22]. Matlab is the main software to collect traffic flows step by step, and to recall control algorithms based on the current states. After time settings for the next cycle are evaluated, they are sent back to Vissim simultaneously. The simulation results are logged in Excel files by Vissim.

Two cases are setup in Vissim, 1) Case I: Marina Bay Area (MBA) is a closed road network locating in Central Business District (CBD) of Singapore (Fig. 1). There are 14 junctions in total and 2 - 4 phases specified for each junction. The simulating period is one hour from 08 : 00 am to 09 : 00 am, since traffic flows during this period are largest during a day.

In this case, the vehicle inputs are identical and variations of traffic flows are not considered.

2) Case II: Clementi is a open road network, which has quite large traffic demands. There are only 5 junctions in Clementi and one arterial road considered (Fig. 5). The simulating period is 15 hours from 07 : 00 am to 10 : 00 pm, which includes most of heavy traffic period.

Fig. 5. Clementi in Singapore

In this case, variations of traffic flows are considered to verify the robustness of IT strategy. There are 10 scenarios with variation of around 15% simulated one by one sequentially.

B. Criteria of Comparison

For these two cases, fixed-time algorithm, SCATS-like strategy, and IT strategy are applied for comparisons. Fixed-time algorithm is obtained based on Webster algorithm [1], which is also the initial settings of IT strategy. SCATS is used in Singapore, but not released in public. A so-called SCATS-like algorithm is used here [23].

Criteria of Comparison in this paper include Total Number of Vehicle (TNV) and Average Delay Time (ADT). Delay time is the subtraction between total travel time of all vehicles and total free-flow travel time (travel time when vehicles are running in free-flow speed). TVN stands for the total number of vehicles which have left the network. ADT means the delay time per vehicle.

C. Origin-Destination Pairs

Origin-Destination (OD) pairs [24] are mainly designed to construct traffic flows close to the real situations on working days since every vehicle has its own origin and destination. In simulation areas, origin and destination points are located at the margins and parking lots inside the networks. LSQR is carried out to find OD matrices with the objective that estimated traffic flows \( X^* \) are close to real data \( X \). Root Mean Square Error (RMSE) in Eq. (13) is calculated to quantify the total percentage error of OD estimate. In Vissim, files *.weg and *.bew store the paths, volumes and costs between origins and destinations, which are created under fixed-time algorithm. These files guarantee that traffic conditions are completely identical for all algorithms.

\[
RMSE = \sqrt{\frac{1}{NM} \sum_i \sum_{\tau} (x_i^*(\tau) - x_i(\tau))^2} \times 100\% \] (13)

where \( i \) is the index of lane groups, \( \tau \) is the index of time intervals; \( N \) is total number of lane groups; \( M \) is total number of time intervals.

1) Case I: In MBA, there are 13 origins and 12 destinations included. Time interval is set to be 10 minutes. OD matrices are estimated based on average traffic flows within 1 month’s working days. \( RMSE \) of OD estimation is 17.83%.

2) Case II: In Clementi, in order to estimate traffic flows more accuracy, each point going in or out of the network are modeled as origin or destination. Therefore 14 origins and 15 destinations are included. \( RMSE \) of OD estimation is 5.38%.

The same to the procedures of Case I, initial OD matrices \( \hat{OD} \) are obtained. Then ten sets of OD matrices \( OD_v \) with variations are calculated in Eq. (14).

For each time interval,

\[
\hat{OD}_v(\tau) = \varphi \cdot OD(\tau) \cdot rand + OD(\tau)
\] (14)

where \( \varphi \) is the amplifier factor; \( rand \) is matrix of random number with proper dimension, and each item in \( rand \in [-1, 1] \); \( \cdot \) is inner product.
V. SIMULATION RESULTS

In this section, simulation results are summarized. Some parameters are described first.

- As described in Eq. (4), \( \alpha \) and \( \beta \) are set to be 0.8 and 0.2, respectively.
- Total Phase Difference \( \theta \), as shown in Eq. (15), is used to indicate the convergence of phase time for every iteration.
\[
\theta = \sum_{k} \sum_{j \in J} \sum_{p \in P_j} \| u_{j,p,k}(k) - u_{j,p,h}(k) \| \tag{15}
\]

where \( u_{j,p,h}(k) \) is historical phase duration.
- Average delay time reduction \( I_{ADT} \) for the whole network is calculated as Eq. (16):
\[
I_{ADT} = \frac{ADT_{FT} - ADT_{IT}}{ADT_{FT}} \times 100\%; \tag{16}
\]

where \( ADT_{FT} \) is the ADT under fixed-time strategy; \( ADT_{IT} \) is ADT under all strategies.

A. Case I

Iterative tuning (IT) is applied in the simulations. For each iteration, one set of OD matrices are simulated and traffic dynamics are completely identical. In the first iteration, historical phase time \( U_h \) and historical traffic flows \( X_h \) are obtained from fixed-time strategy. Based on Eq. (8), phase time \( U \) is tuned iteratively. In the first few iterations, \( \theta \) is going up since traffic flow are tuned. When traffic flows are tuned well, \( \theta \) is approaching 0 gradually (Fig. 6), which indicates tuned phase times will approach steady states gradually.

![Fig. 6. Total Phase Difference](image1)

![Fig. 7. Average Delay Time](image2)

As simulations are carried on iteratively, average delay time decreases until steady states. The delay time versus number of iterations are shown in Fig. 7. The red line in the graphs indicates that final steady state of ADT is 50.29 seconds. The trajectories show that after around 30 iterations, the delay time is reduced to the optimum point and goes to steady states after 60 iterations.

Simulation results under fixed-time Strategy, SCATS-like and IT are summarized in Table I. At the end of simulation, there is no traffic congestion, TNV under all strategies are similar. From data of ADT, IT strategy can obtain much better performance over other strategies. Comparing with SCATS, IT strategy can outperform in average delay time reduction by 7.34%.

From traffic raw data analysis, junction 6 is critical junction in Marina Bay Area. Traffic congestion is formed around Junction 6 due to high traffic demands. Maximum queue lengths within a cycle and corresponding phase durations are depicted in Fig. 8 for junction 6, respectively.

The horizontal axis represents the number of cycles. In one hour simulation, there are 45 cycles in total. For junction 6 (Fig. 8), under fixed-time strategy queue is accumulated from 25\textsuperscript{th} cycle to 35\textsuperscript{th} cycle during Phase I. Correspondingly under IT strategy, more green time is allocated to Phase I and queue is rapidly discharged. Meanwhile, the green time of Phase III is reduced and a small queue is formed. The comparison shows that IT strategy tunes phase splits based on prevailing traffic demands. Queue length and queue accumulating period are largely reduced.

![Fig. 8. Comparisons of Junction 6](image3)

B. Case II

In the simulation of Clementi, variations of traffic flows are considered. There are 10 scenarios designed for simulations. Under fixed-time strategy, average coefficient of variation of traffic flows is 18.03%. Iterative Tuning strategy is applied to these 10 scenarios one by one sequentially. Total phase difference is shown in Fig. 9. In the first iteration, traffic signal schedules and traffic flows are obtained from fixed-time strategy and average historical data, respectively. In total there are 200 iterations and 20 batches of iterations for these ten scenarios. Like the results in Case I, \( \theta \) is going up and falls down with small oscillations eventually. Meanwhile, as shown in Fig. 10, average delay time is reduced gradually until steady states.

Ten scenarios in the last batch of iterations under IT strategy are compared with fixed-time strategy (Fig. 11). For all scenarios, IT strategy outperforms fixed-time considerably. For most of scenarios, delay times under IT strategy are also lower than that under SCATS-like strategy.
Average delay time for all strategies are summarized in Table II. Since after 10 pm, traffic demand is quite small and there is no congestion everywhere, TNV is very similar. During entire simulation period, IT strategy can reduce the delay time by 18.38%. Due to variations of traffic flows, IT strategy can not response to the disturbances and the performance is affected slightly. However, the performance is still better than that of SCATS by around 3.26%, which proves that IT strategy can work very well on working days.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>TNV (veh)</th>
<th>ADT (s/veh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed-Time</td>
<td>33995</td>
<td>68.8456</td>
</tr>
<tr>
<td>SCATS-like</td>
<td>33992</td>
<td>58.4588(15.12%)</td>
</tr>
<tr>
<td>IT</td>
<td>34000</td>
<td>56.1943(18.38%)</td>
</tr>
</tbody>
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VI. CONCLUSIONS

This paper introduces Iterative Tuning (IT) strategy with phase splits by using historical traffic flows. The analysis of historical raw data shows that traffic flow patterns on weekdays are repetitive on a weekly basis with small variations. With anticipation of traffic flows, IT controllers of all junctions tune daily traffic signal schedules iteratively to reduce delay time of traffic network. The simulation results of Marina Bay Area show that IT strategy can reduce the delay time of the network considerably. Although IT strategy can not response to the non-repetitive disturbances, from the simulations of Clementi with non-repetitive disturbances, it still works well for working days as variations of traffic flows are quite small.

The future work will focus on Iterative Tuning Strategy with cycle time, offset and splits, which can work cooperatively to approach the suitable traffic signal schedules.

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