Mechanisms for Arranging Ride Sharing and Fare Splitting for Last-Mile Travel Demands

(Extended Abstract)

Shih-Fen Cheng  Duc Thien Nguyen  Hoong Chuin Lau
School of Information Systems  Singapore Management University
{sfcheng,dtnguyen.2011,hclau}@smu.edu.sg

ABSTRACT
A great challenge of city planners is to provide efficient and effective connection service to travelers using public transportation system. This is commonly known as the last-mile problem and is critical in promoting the utilization of public transportation system. In this paper, we address the last-mile problem by considering a dynamic and demand-responsive mechanism for arranging ride sharing on a non-dedicated commercial fleet (such as taxis or passenger vans).

Our approach has the benefits of being dynamic, flexible, and with low setup cost. A critical issue in such ride-sharing service is how riders should be grouped and serviced, and how fares should be split. We propose two auction designs which are used to solicit individual rider’s willing payment rate and compensation rate (for extra travel, if any). We demonstrate that these two auctions are budget balanced, individually rational, and incentive compatible. A series of experimental studies based on both synthetic and real-world datasets are designed to demonstrate the pros and cons of our two proposed auction mechanisms in various settings.

Categories and Subject Descriptors
H.4 [Information Systems Applications]: Miscellaneous

General Terms
Economics, Algorithms

Keywords
ride sharing, mechanism design, cost sharing

1. INTRODUCTION
In most urban cities, building and promoting the use of public transport is the only way to satisfy demands from the ever-increasing city population. Improving the accessibility and attractiveness of public transport is thus an important policy goal for most city planners. In this paper, we study the design of ride-sharing-based last-mile services for cities with well-established public transports and assume that there is a fleet of committed service vehicles

who are willing to serve ride-sharing passengers at predetermined distance-based fare. Although these service vehicles can come from any sources, vacant taxis or passenger vans would be the best candidates, as serving last-mile demands also help to alleviate overcapacity during off-peak hours.

All last-mile demands should depart from the same hub at the same time, and should be reported sufficiently earlier than their estimated arrival times at the hub. Demands are then assigned to available individual service vehicles, where service sequences and corresponding payments are computed for all demands.

To facilitate such last-mile service, we need to address two important classes of closely related problems. The first problem is to assign last-mile demands to individual vehicles and to determine service orders. The second problem is to determine how fares should be split among riders, considering both distances traveled and inconveniences caused by using ride-sharing services. The first problem is similar to the well-known para-transit problem [2] and dial-a-ride problem [1], and can be formulated and solved similarly; the solution for the second problem, on the other hand, is not so obvious as private information (e.g., desired meter rate) from individual riders is needed. To solicit reliable private information from individuals, we need to design mechanisms that are guaranteed to be incentive compatible and implementable.

2. MECHANISM DESIGN FOR PROBLEMS WITH SPATIAL REQUIREMENTS
Our proposed mechanisms are different from classical ones found in literature since agents would value their received services based on allocation itself and extra travel required. This implies that agent’s utilities depend not just on their own choices, but also on other agents’ choices and allocations. To reflect this special property, we define agent’s utility function in the ride-sharing context as follows:

\[ u_i = (m_i s_i - m_i (B_i - s_i)) - (m^* s_i - m^* (B_i - s_i)) \],

where \( m_i \) is agent i’s private payment and compensation rate for distance traveled, \( B_i \) is agent i’s real distance traveled, \( s_i \) is agent i’s direct travel distance, and \( m^* \) is the rate actually charged. The first part of \( u_i \) reflects how agent i would value any allocation, and the second part of \( u_i \) represents the payment agent i needs to pay.

With the utility function defined in (1), we are now ready to define our mechanism, which is essentially an auction capable of handling both the price and spatial information.
A bid acceptable by our auction mechanism is in the form of \((s_i, m_i)\), where \(s_i\) denotes both agent \(i\)'s destination and the direct travel distance to reach it (from the hub), and \(m_i\) denotes agent \(i\)'s desired meter rate.

As a DARP-style routing problem needs to be solved for each winner determination instance, exact formulation would not be scalable enough to meet the short turnaround time given (usually less than 10 minutes in practice). To address such limitation, we propose two approximate mechanisms that are scalable while at the same time still demonstrate desirable properties such as incentive compatibility and budget balance. In the interest of space, we highlight only the first mechanism.

### 2.1 The Bottom-up Mechanism

The formal definition of the clearing rule of the bottom-up mechanism can be found in Algorithm 1. The inputs required by the Algorithm 1 include \(N\), which denotes the set of all riders, and \(\{(s_i, m_i)\}_{i \in N}\), which represents the collection of bids for all riders in \(N\).

The bottom-up mechanism first solves \textit{Program-A}\(^1\) to obtain an assignment and routing plan. For each solution obtained, we check whether the lowest paying customer can afford the returned meter rate (line 4). If not, the lowest paying customer is removed from the set \(N\) (line 6); the lower bound on the meter rate \((m_i)\) is also updated accordingly (line 5). The above process repeats until a solution that satisfies all individual payment constraints is found. At termination, the bottom-up mechanism reports the set of riders to be served \((S)\), how riders in \(S\) are grouped (denoted as \(k_i\), which represents the identity of the service vehicle assigned to rider \(i\)), the service order in the group (denoted as \(a_i\)) and the meter rate \((m^*)\) to be paid by all served customers.

**Algorithm 1:** The bottom-up mechanism.

<table>
<thead>
<tr>
<th>Input: ((N, {(s_i, m_i)}_{i \in N}))</th>
<th>Output: ((S, {(k_i, a_i)}_{i \in S, m^*}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (S \leftarrow \emptyset, m_1 = 0)</td>
<td>2 \textbf{while} (N \neq \emptyset) \textbf{do}</td>
</tr>
<tr>
<td>3 ({(S, {(k_i, a_i)}_{i \in S, m^*}) \leftarrow \text{PROGRAM-A}(N, m_1))</td>
<td>4 \textbf{if} (\min_{i \in S} m_i &lt; m^*) \textbf{then}</td>
</tr>
<tr>
<td>5 (m_1 = \min_{i \in S} m_i)</td>
<td>6 (N \leftarrow N \setminus {\arg\min_{i \in S} m_i})</td>
</tr>
<tr>
<td>7 \textbf{else}</td>
<td>8 (N \leftarrow \emptyset)</td>
</tr>
<tr>
<td>9 \textbf{end}</td>
<td>10 \textbf{return} ((S, {(k_i, a_i)}_{i \in S, m^*}))</td>
</tr>
</tbody>
</table>

### 2.2 Important Properties

The bottom-up mechanism can be shown to be \textit{budget balanced, individually rational, and incentive compatible}. Budget balance is built in as a constraint in \textit{Program-A}. Individual rationality is embedded in Algorithm 1, since riders with \(m_i < m^*\) in all intermediate allocations are removed.

The incentive compatibility result can also be proved as we can easily show that an agent would gain nothing by lying about his true value.

### 3. NUMERICAL EXPERIMENTS

The performance of our proposed mechanisms are tested using both synthetic and real-world-inspired datasets. For all cases, we measure the following four metrics: 1) number of riders, 2) sum of direct-travel distance served, 3) distance saved due to shared rides, and 4) rider’s surplus. The real-world-inspired dataset is based on the last-mile travel demands captured by the EZ-Link payment system (http://en.wikipedia.org/wiki/EZ-Link) in Singapore. The EZ-Link payment system is an electronic payment system that can be used for all modes of public transportation in Singapore. Its penetration is almost universal among residents, and as such, we can identify travelers who alight at a major station and go on to make short travels to nearby neighborhoods (via buses or light-rails). We focus on one of the busiest train station in Singapore, Ang Mo Kio. The distribution of last-mile demands can be found in Figure 1, with color and size of circle showing different levels of demand concentrations. The black square near the center of the map is the hub.

From all numerical results, we can see that in general the bottom-up mechanism dominates the raising-cost mechanism (the other approximate mechanism not included due to space limit) in the first three metrics, while the raising-cost mechanism produces better rider’s surplus.

### 4. REFERENCES


---

\(^1\)\textit{Program-A} is an integer linear programming model used to compute all the intermediate outputs, and is a variant of the well-known DARP model. Due to the space limit, the complete mathematical model is not included.